807 AP Physics I
Name: _____

Schmidt
Block: __

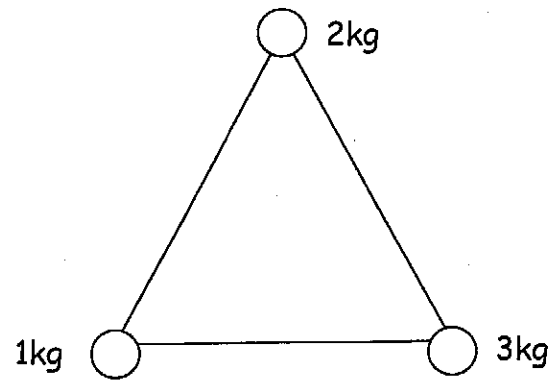
U6H1 **Center of Mass (Sec. 12.2) Discrete Systems)**

$$x_{com} = \frac{\sum_i m_i x_i}{\sum_i m_i};\ y_{com} = \frac{\sum_i m_i y_i}{\sum_i m_i};\quad z_{com} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

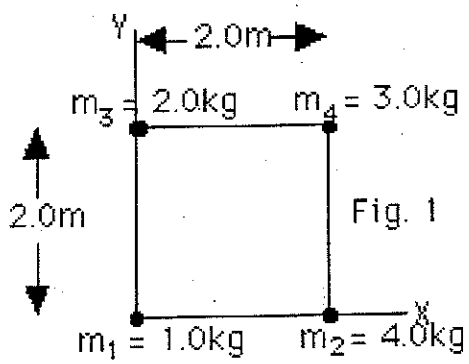
Type I problems: A collection of discrete point objects

1. Two particles of 2kg and 4kg are placed at (1,4) and (3, -2) in x and y plane. If the coordinates are in m, then find the position of COM. (7/3, 0)

2. Three particles of mass 1kg, 2kg, 3kg are positioned at the vertices of an equilateral triangle of side 1 m as shown in the figure. Find the center of mass of the particle (in m)
(2/3, 0.29)



3. Four particles are distributed in the X-Y plane as shown in Fig below. Find the coordinates (X_{cm}, Y_{cm}) of the particles. The particles are on the corners of a 2.0-m square.
(1.4, 1)



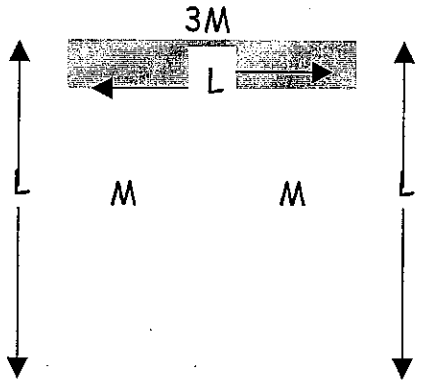
4. Find the position of the center of mass of five equal-mass particles located at the five corners of a square based right pyramid with sides of length L and height h.

Type II Problems: Object made of a composite of uniform bodies

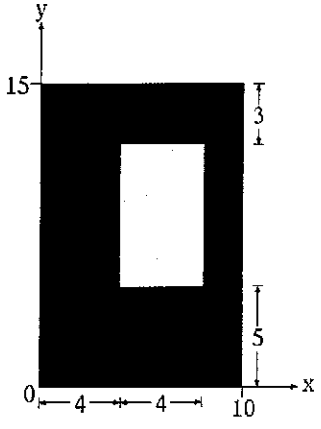
- Center of Mass is at the geometrical center of uniform bodies (bodies of uniform density).
- For objects which are made up of a collection of uniform bodies (of mass m_i and center of mass (x_i, y_i, z_i)) the center of mass is given by

$$x_{com} = \frac{\sum_i m_i x_i}{\sum_i m_i};\ y_{com} = \frac{\sum_i m_i y_i}{\sum_i m_i};\ z_{com} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

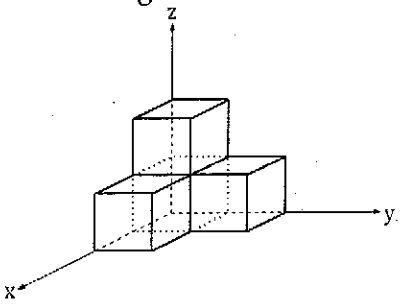
1. Three thin rods each of length L are arranged in an inverted U as shown. The two rods on the arms of the U each have mass M; the third has mass 3M. Where is the center of mass of the assembly? (0.2L beneath center of 3M)



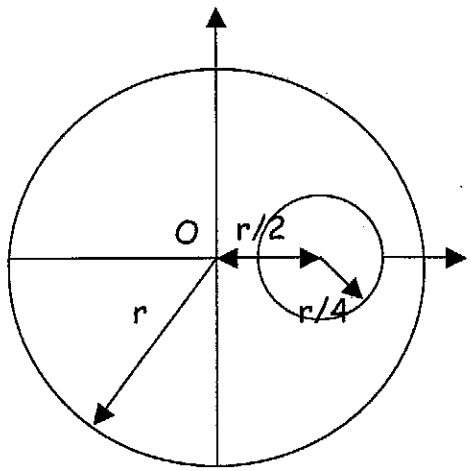
2. Find the center of mass of the following uniform structure (see figure) using the lower left corner as an origin. (4.77cm, 7.27cm)



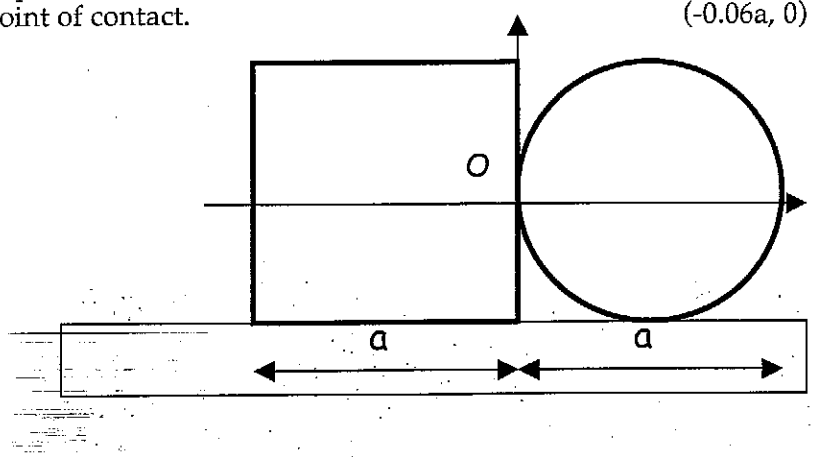
3. Find the center of mass (x,y,z) of the following structure of uniform, identical cubes if the length of each side of a cube is 1. (3/4, 3/4, 3/4)



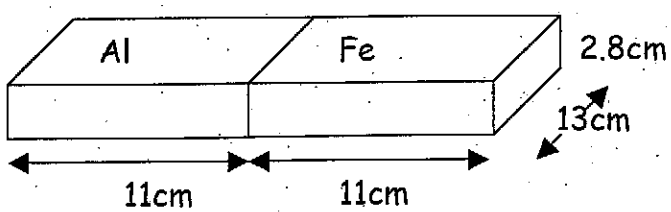
4. A small circular portion of radius r/4 is taken out from a circular disc of uniform thickness having radius r and mass m as shown. Determine the center of mass of remaining portion of the disc. (- r/30)



5. A circular and a square plate are placed in contact as shown in the figure. If the material and thickness of the two plates are the same, then find the COM of the system of bodies as measured from the point of contact. (-0.06a, 0)



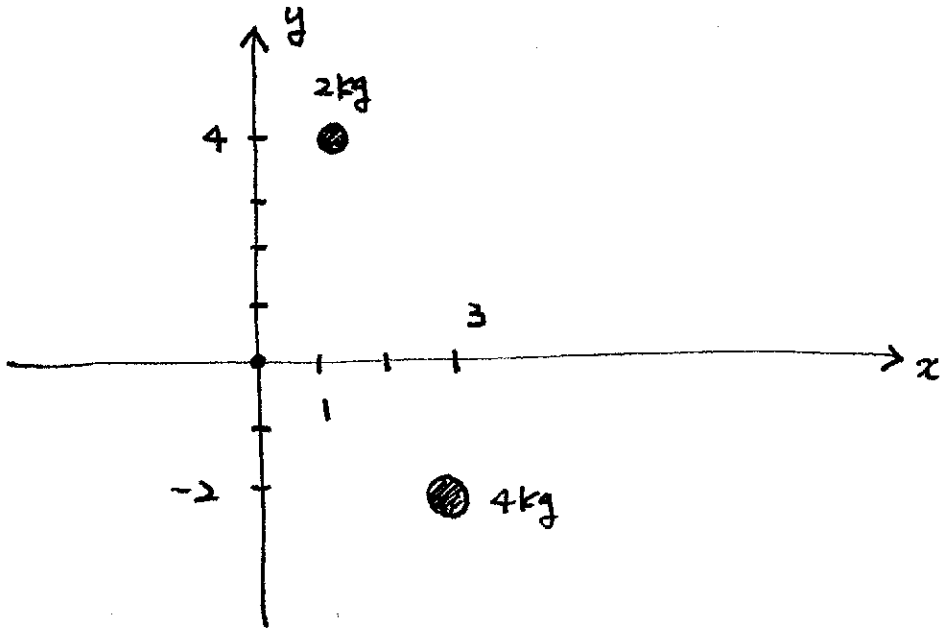
6. Shown below is a composite slab; half of the slab is made of aluminum (density 2.7 g/cm³) and the other half made of iron (density = 7.85 g/cm³). Find the COM of the slab. (in Iron; midheight, midwidth, 2.7 cm from midlength)



U6H1

Type I.

1.



x- direction.

$(2+4)\cdot x = 2\times 1 + 4\times 3$

$x = \frac{2+12}{6}$

$= \frac{7}{3}$

y - direction.

$(2+4)y = 2\times 4 + 4\times -2$

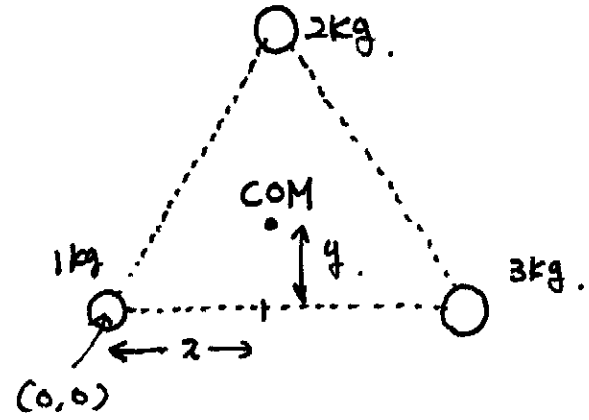
$y = \frac{8-8}{6}$

$= 0$.

U6H1

Type I

2.



x-direction.

$(1+3+2)\cdot x = 0+3\times 1+2\times\frac{1}{2}$

$x = \frac{3+1}{6}$

$= \frac{2}{3}$

y-direction.

$(1+3+2)y = 0+0+2\times\frac{1}{2}\sqrt{3}$

$y = \frac{\sqrt{3}}{6}$

LJ6H1

Type I

3. x- direction

$$(1+4+3+2)\, x_{CM} = 0 + 4\times 2 + 3\times 2 + 0$$

$$x_{CM} = \frac{8+6}{10}$$

$$= \frac{7}{5}$$

y- direction

$$(1+4+3+2)\, y_{CM} = 0 + 0 + 3\times 2 + 2\times 2$$

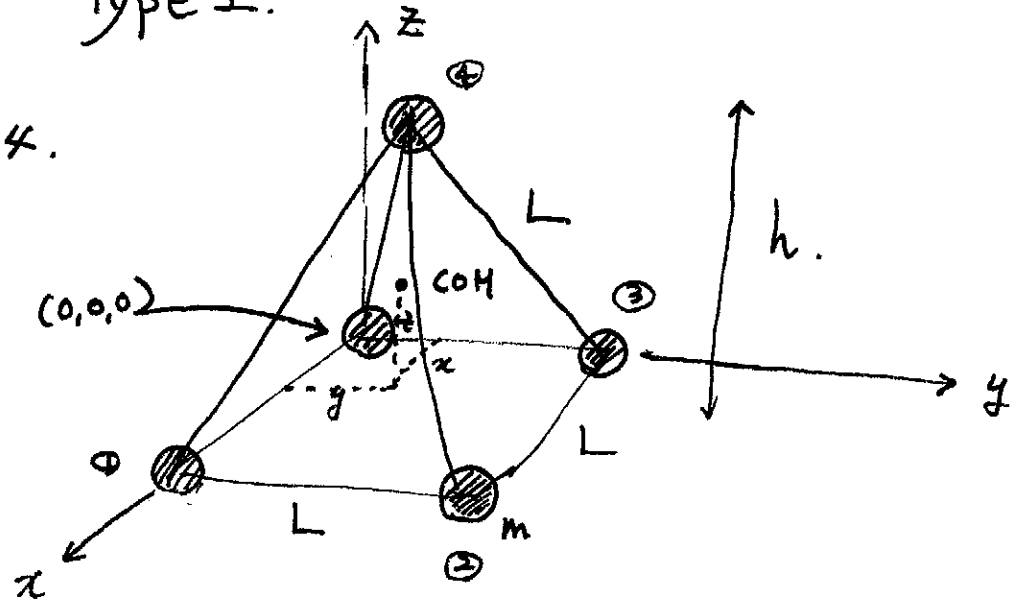
$$y_{CM} = \frac{6+4}{10}$$

$$= 1.$$

W6H1

Type I.

4.



x- direction

$$5m \cdot x = m \cdot L + mL + 0 + m\frac{L}{2}$$

$$x = \frac{\frac{5}{2}L}{5} = \boxed{\frac{L}{2}}$$

y-direction

$$5m \cdot y = 0 + m \cdot L + m \cdot L + m\frac{L}{2}$$

$$y = \boxed{\frac{L}{2}}$$

z-direction

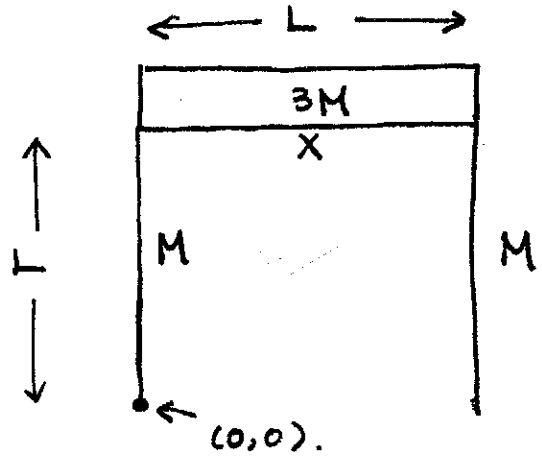
$$5m \cdot z = 0 + 0 + 0 + m\frac{h}{2}$$

$$z = \boxed{\frac{h.}{10}}$$

LL6H1

Type II

1.



x – direction

sym. $\rightarrow$ $x = \frac{L}{2}$.

y – direction

$$(M + M + 3M)\,y = \frac{L}{2}M + \frac{L}{2}M + L \cdot 3M$$

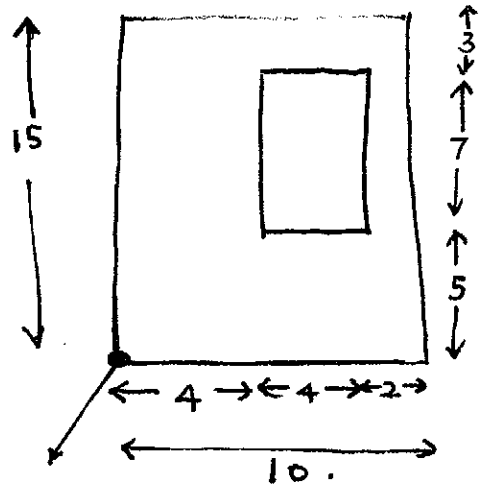
$$y = \frac{4L}{5}$$

$$= \frac{4}{5}L$$

LIGHT

Type II

2.



(0,0,0)

x-direction.

$$(15\times10 - 7\times4)\,\bar{x} = 15\times10\times5 - 7\times4\times6$$

$$\bar{x} = \frac{750 - 168}{122} = 4.77.$$

y- direction

$$(15\times10 - 7\times4)\,\bar{y} = 15\times10\times\frac{15}{2} - 7\times4\times\left(5+\frac{7}{2}\right)$$

$$\bar{y} = \frac{1125 - 238}{122} = 7.27$$

U6H1

Type II

3. x – direction

$$4m \cdot x = m \cdot \frac{L}{2} + m \frac{3L}{2} + m \frac{L}{2} + m \frac{L}{2}$$

$$x = \frac{3L}{4} = \frac{3}{4}.$$

y – direction.

$$4m \cdot y = m \frac{1}{2} + \frac{1}{2}m + \frac{3}{2}m + \frac{1}{2}m.$$

$$y = \frac{3}{4}$$

z – direction.

$$4m \cdot z = m \frac{1}{2} + m \frac{1}{2} + \frac{1}{2}m + \frac{1}{2}3m$$

$$z = \frac{3}{4}$$

LU6H1

Type II.

4. y-direction Symmetry.

x-direction

$$\left(\pi r^2 - \pi\left(\frac{r}{4}\right)^2\right)\bar{x} = \pi r^2 \times 0 - \pi\left(\frac{r}{4}\right)^2 \cdot \frac{r}{2}.$$

$$\frac{15\pi r^2}{16}\bar{x} = -\frac{\pi r^3}{16 \cdot 2}$$

$$\bar{x} = -\frac{r}{30}$$

UGH1

Type II.

5. y – direction $\longrightarrow$ Sym $(y = 0)$

x – direction

$$\left(a \cdot a + \pi \left(\frac{a}{2}\right)^2\right) x = a \cdot a \cdot \left(-\frac{a}{2}\right) + \pi \left(\frac{a}{2}\right)^2 \cdot \left(\frac{a}{2}\right)$$

$$\left(a^2 + \frac{\pi}{4} a^2\right) x = -\frac{a^3}{2} + \frac{\pi a^3}{8}$$

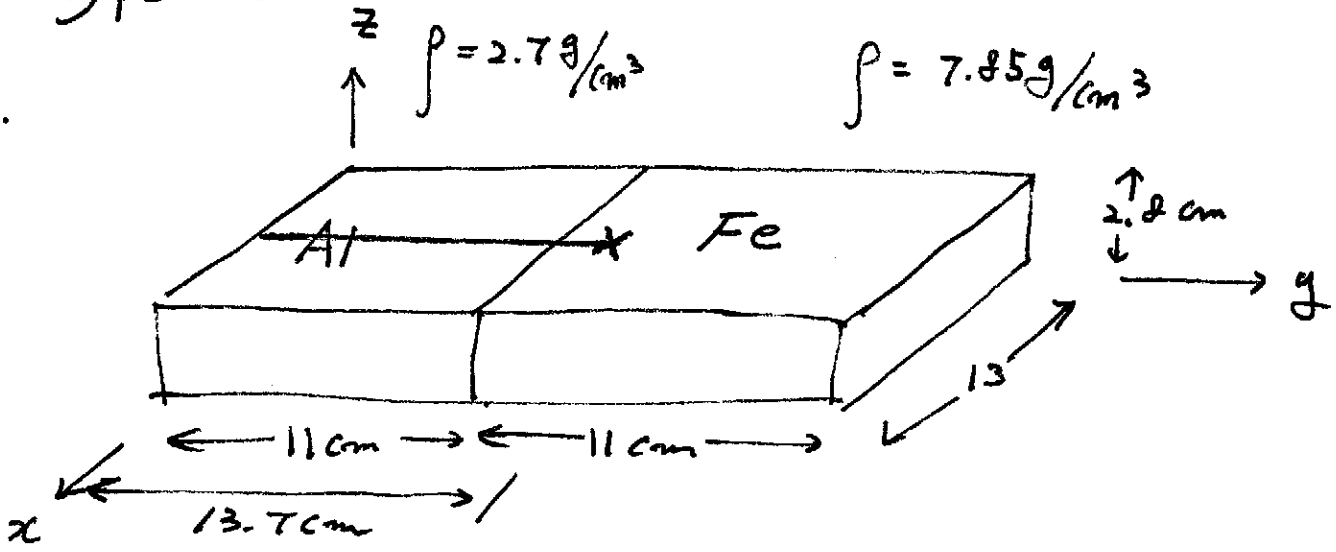
$$x = \frac{-\frac{1}{2} + \frac{\pi}{8}}{1 + \frac{\pi}{4}} a$$

$$= -0.06a$$

U6H1

Type II.

6.



x and z are sym. $x_{CM} = \frac{13}{2}$ $z_{CM} = \frac{2.8}{2}$

y-direction

$(2.7 \times 11 \times 13 \times 2.8 + 7.85 \times 11 \times 13 \times 2.8)\, y$

$= 2.7 \times 11 \times 13 \times 2.8 \times \frac{11}{2} + 7.85 \times 11 \times 13 \times 2.8 \times (11 + \frac{11}{2})$

$4224.2 \cdot y = 57808.$

$y = 13.685 \text{ cm}.$