

Name: _____

Block: _____

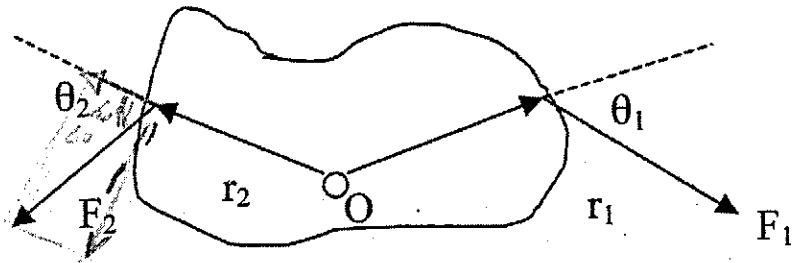
U6H4

Rotation (Dynamics)

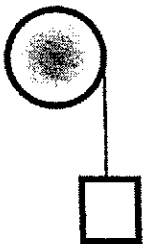
Defined as $\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta$ (θ is the angle between \vec{r} and \vec{F} when placed tail to tail)
Newton's Second Law for Rotation: $\vec{\tau} = I\vec{\alpha}$; $\vec{\alpha}$ may be positive or negative

Torque is a vector quantity. In rotation, conventionally, clockwise is considered negative and counterclockwise positive. Torque is considered negative if the force makes an object rotate clockwise starting from rest and positive if it makes an object rotate counterclockwise starting from rest.

1. The object in the figure below is pivoted at the point O. If $r_1 = 1.30\text{m}$, $r_2 = 2.25\text{m}$, $F_1 = 4.20\text{ N}$, $F_2 = 4.90\text{N}$, $\theta_1 = 75^\circ$, and $\theta_2 = 60^\circ$, what is the net torque about the pivot? (4.28 Nm)



2. A thin spherical shell has a radius of 1.90m. An applied torque of 960 Nm imparts to the shell an angular acceleration of 6.20 rad/s^2 about an axis through the center of the shell. (a) What is the rotational inertia of the shell about the axis of rotation? (b) Calculate the mass of the shell. (155kg m², 64.4kg)
3. A mass $m = 400\text{g}$ hangs from the rim of a wheel of radius $r = 15\text{cm}$. When released from rest, the mass falls 2.0 m in 6.5 s. Find the moment of inertia of the wheel. ($I = 0.94\text{ kg m}^2$)

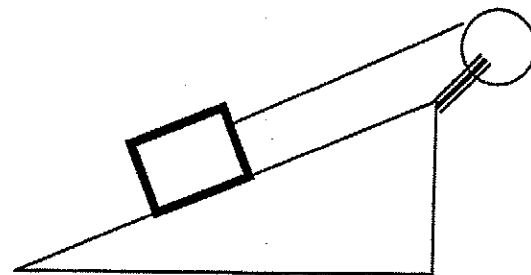


4. In an Atwood's machine, one block has mass of 5kg, and the other mass of 3kg. The pulley, which is mounted on horizontal frictionless bearings, has a radius of 10cm. When released from rest, the heavier block is observed to fall 1m in 2.0s (without the cord slipping over the pulley). (a) What is the acceleration of each block? (b) What is the tension in the part of the rope that supports the heavier block? (c) What is the tension in the part of the rope that supports the lighter block? (d) What is the angular acceleration of the pulley? (e) What is its rotational inertia? (0.5m/s², 31.5 N, 47.5 N, 5rad/s², 0.32 kg m²)

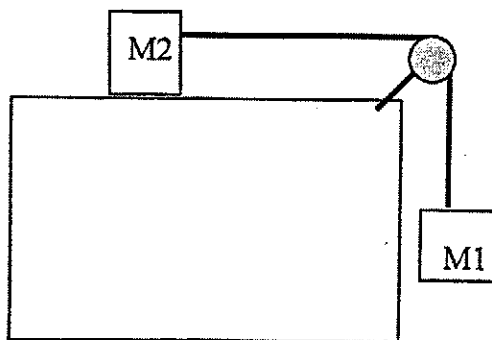
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5. A wheel (disk) of radius 0.20 m and mass 1kg is mounted on a frictionless horizontal axis. A massless cord is wrapped around the wheel and attached to a 2.0kg object that slides on a frictionless surface inclined at an angle of 60° with the horizontal. What is the acceleration of the block on the incline as well as the angular acceleration of the wheel about its axis of rotation? (6.93 m/s^2 , 34.64 rad/s^2)



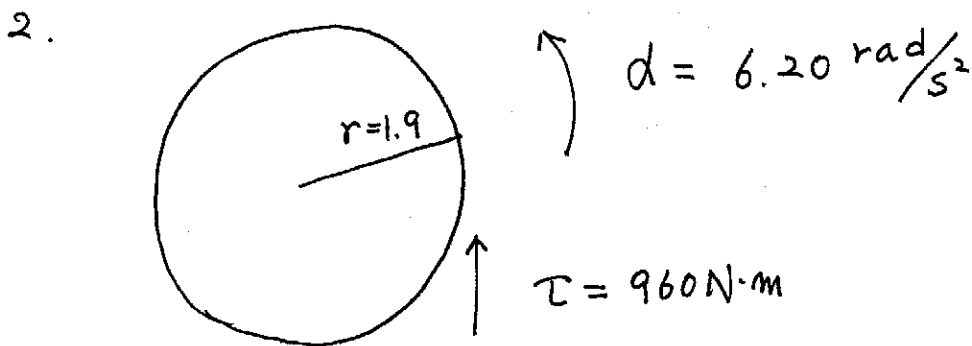
6. Blocks m_1 and m_2 are connected by a massless string that passes over a pulley (disk) which turns on frictionless bearings. The blocks are released from rest. Suppose the pulley has a mass M and a radius R and is shaped like a disk. Find the expression for the acceleration 'a' of the blocks and the tensions in the horizontal and vertical sections of the string, T_h and T_v respectively. (What is the acceleration, T_h and T_v for a system where $m_1 = 1\text{kg}$, $m_2 = 2\text{kg}$ and pulley has a mass of 0.5kg and radius 0.1m). $(a = \frac{m_1 g}{\frac{1}{2}M + m_2 + m_1}, a = 3.08 \text{ m/s}^2, T_h = 6.15 \text{ N}, T_v = 6.92 \text{ N})$



7. In an Atwoods machine setup, the two hanging masses are 4kg and 2kg respectively. The pulley shaped like a disk is 6cm in radius and has a mass of 2kg. As the pulley turns, friction at the axle exerts a torque of 0.5Nm. If the blocks are released from rest with the 4kg block 1m from the floor, how long does it take the 4kg block to reach the floor? (1.09 s)

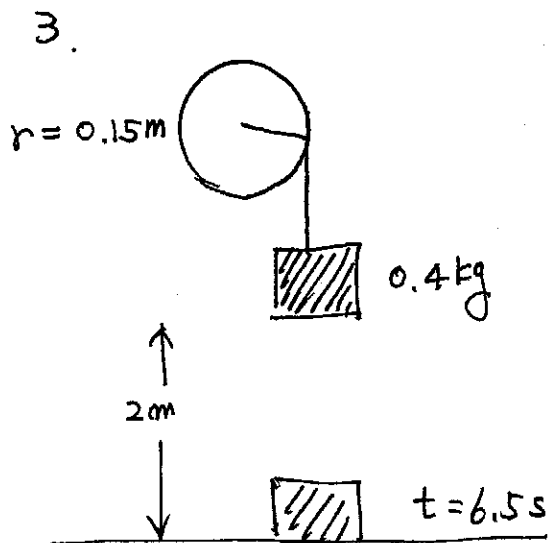
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1.
$$\tau = -1.3 \times 4.2 \times \sin 75^\circ + 2.25 \times 4.9 \times \sin 60^\circ$$
$$= 4.274 \text{ Nm}$$



(a) $\tau = I \cdot \alpha \rightarrow I = \frac{\tau}{\alpha} = \frac{960}{6.2} = 154.8 \text{ kg}\cdot\text{m}^2$

(b) $I = \frac{2}{3} MR^2 \rightarrow M = \frac{3I}{2R^2} = 64.3 \text{ kg}$



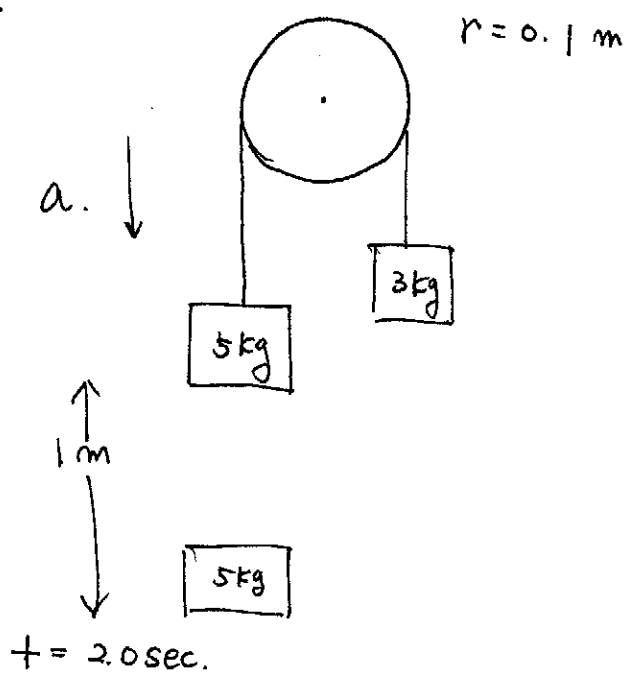
$$y = v_0 t + \frac{1}{2} a t^2$$
$$2 = \frac{1}{2} \cdot a \cdot (6.5)^2 \rightarrow a = 0.095 \text{ m/s}^2$$

$$\alpha = \frac{a}{r} = 0.63 \text{ rad/s}^2$$

$$\tau = F \cdot r = I \cdot \alpha$$

$$I = \frac{F \cdot r}{\alpha} = \frac{4 \times 0.15}{0.63} = 0.95 \text{ kg}\cdot\text{m}^2$$

4.



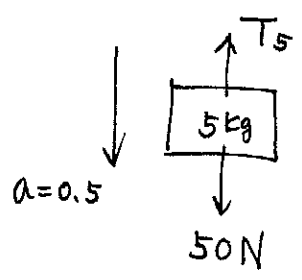
(a)

$$y = v_0 t + \frac{1}{2} a t^2$$

$$1 = \frac{1}{2} \cdot a \cdot (2)^2$$

$$a = 0.5 \text{ m/s}^2$$

(b)

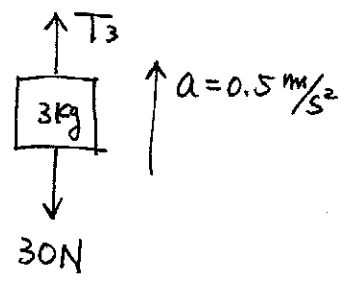


$$F = ma.$$

$$50 - T_5 = 5 \times 0.5$$

$$T_5 = 47.5 \text{ N}$$

(c)



$$T_3 - 30 = 3 \times 0.5$$

$$T_3 = 31.5 \text{ N}$$

(d)

$$d = \frac{a}{R} = \frac{0.5}{0.1} = 5 \text{ rad/s}^2$$

(e)

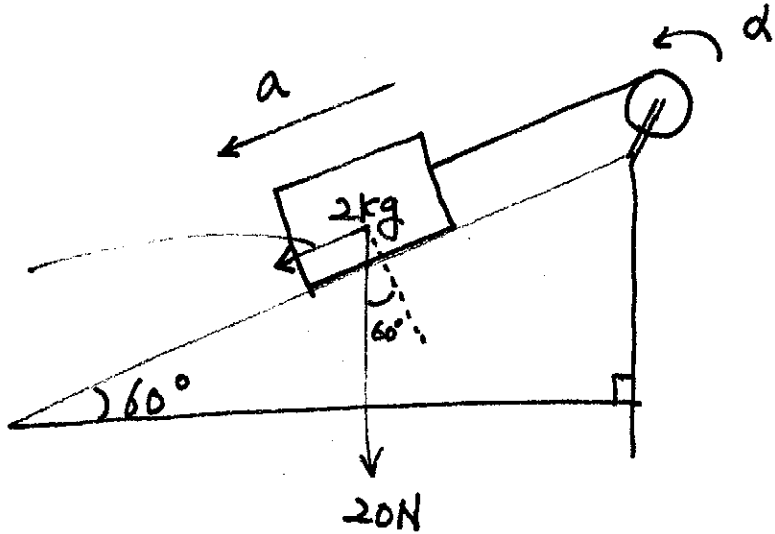
$$\tau = I \cdot d$$

$$I = \frac{\tau}{d} = \frac{47.5 \times 0.1 - 31.5 \times 0.1}{5} = 0.32 \text{ kg m}^2$$

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5. $r = 0.20 \text{ m}$
 $m = 1 \text{ kg}$

$$20 \cdot \sin 60^\circ = 10\sqrt{3}$$



$$I = \frac{1}{2} m r^2$$

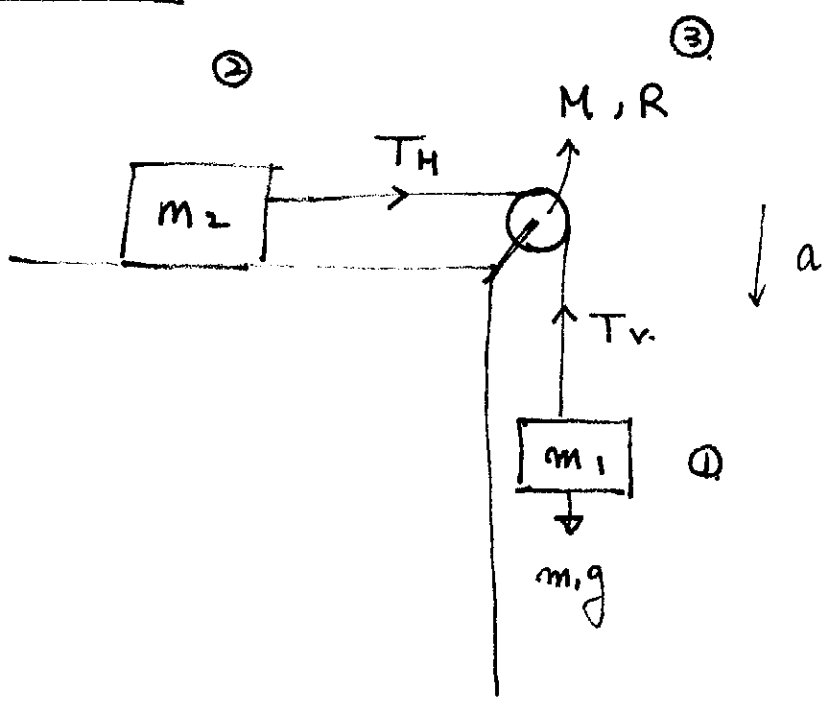
$$F = m a.$$

$$10\sqrt{3} = (2 + \frac{1}{2} \times 1) \cdot a$$

$$\therefore a = 6.93 \text{ m/s}^2$$

$$\alpha = \frac{a}{R} = 34.64 \text{ rad/s}^2$$

6.

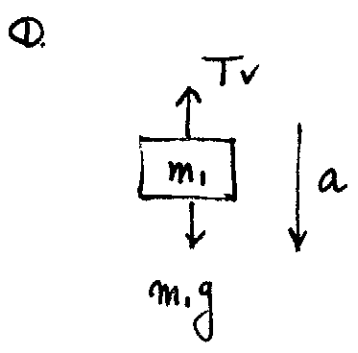


$$F = ma.$$

$$m_1 g = (m_1 + m_2 + \frac{1}{2} M) \cdot a$$

$$a = \frac{m_1 g}{m_1 + m_2 + \frac{1}{2} M}$$

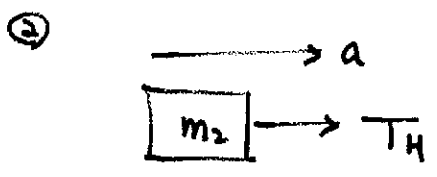
$$\therefore a = \frac{10}{1 + 2 + \frac{1}{2} (0.5)} = 3.08 \text{ m/s}^2$$



$$F = ma$$

$$m_1 g - T_v = m_1 \cdot a.$$

$$T_v = m_1 (g - a) = 1 \cdot (10 - 3.08) = 6.92 \text{ N.}$$



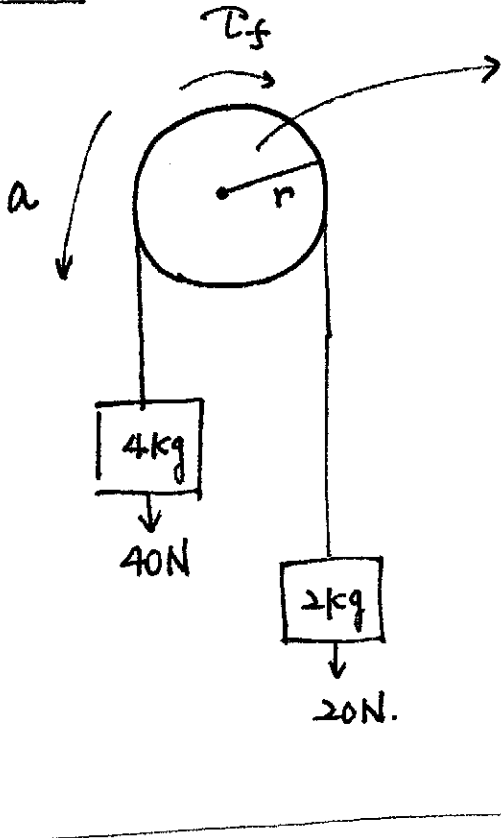
$$F = ma$$

$$T_H = m_2 \cdot a$$

$$= 2 \cdot 3.08 = 6.16 \text{ N.}$$

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7.



$$m = 2 \text{ kg}$$

$$r = 0.06 \text{ m}$$

$$\tau_{\text{friction}} = 0.5 \text{ N}\cdot\text{m}$$

$$y = v_0 t + \frac{1}{2} a t^2$$

$$1 = \frac{1}{2} \cdot a \cdot t^2$$

$$t = \sqrt{\frac{2}{a}}$$

$$F = ma.$$

$$40 - 20 - \frac{0.5}{0.06} = (4 + 2 + \frac{1}{2} \cdot 2) \cdot a$$

$$11.667 = 7 \cdot a.$$

$$a = 1.667 \text{ m/s}^2$$

$$\tau = F \cdot r$$

$$F = \frac{\tau}{r}$$

$$\therefore t = \sqrt{\frac{2}{1.667}} = 1.09 \text{ sec}$$