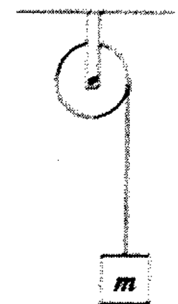
807 AP Physics Schmidt

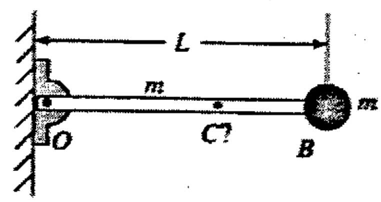
Name: ________________ Block: ______

U6H6 Rotational Dynamics –C.O.E., Work-Kinetic Energy Principle

1. A uniform disk of mass 400 g and radius 12 cm is mounted on a fixed axle. A block whose mass is 50g hangs from a massless cord that is wrapped around the rim of the disk. Find the speed of the falling block after it has descended 50 cm staring from rest? Solve the problem using energy conservation principles. (1.4 m/s)



2. A massless rod of length L has a mass $2m$ fastened at its center and another mass m fastened at one end. On the opposite end from the mass m, the rod is hinged with a frictionless hinge. The rod is released from rest from an initial horizontal position; then it swings down. What is the angular velocity as the rod swings through its lowest (vertical) position? ($\sqrt{\frac{8g}{3l}}$)



3. Consider a rod of length L and mass m which is pivoted at one end, O. The moment of inertia of the rod about O is $I_{rod} = 1/3mL^2$. A very small object with mass m is attached to the other end of the rod at B.

(a) Determine the moment of inertia, I, of the system with respect to the point O. ($4/3\ ML^2$)

(b) Determine the location of the center of mass of the system with respect to the point O. (3/4 L)

(c) Determine the angular velocity of the rod particle system when it is vertical assuming it was released from rest when the rod was horizontal. ($\sqrt{\frac{9g}{4l}}$)

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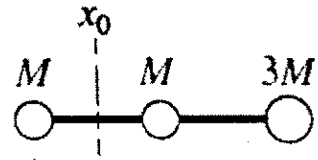
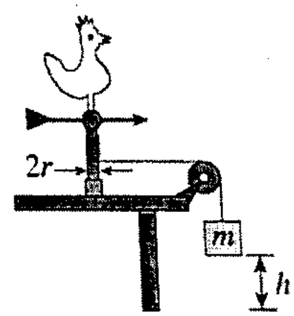
4. The adjacent figure shows a rigid system which can rotate. The mass of each connecting rod is negligible. Treat the masses as particles. The masses are separated by rods of length L, so that the entire length is 2 L.

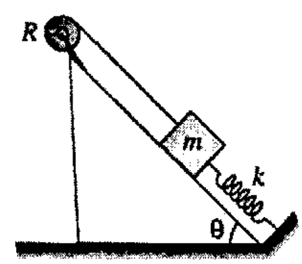


The system is pivoted so that it can rotate about a point $x_0 = L/2$ from the left end. What is the angular velocity of the system when it is vertical? ($\sqrt{\frac{72g}{58l}}$)

5. A bright physics student purchases a weathervane for her father's garage. The vane consists of a rooster sitting on top of an arrow. The vane is fixed to a vertical shaft of radius 0.02m and mass 1.3 kg that is free to turn in its roof mount as shown in the figure. The student sets up an experiment to measure the rotational inertia of the rooster and arrow. String wound about the shaft passes over a pulley and is connected to a 3 kg mass hanging over the edge of the roof. When the 3 kg mass is released, the student determines the time t that the mass takes to fall through a distance 0.7 m. Find the rotational inertia I of the rooster and arrow, if t = 20 s. Answer in units of kgm^2. (3.43 kgm^2)



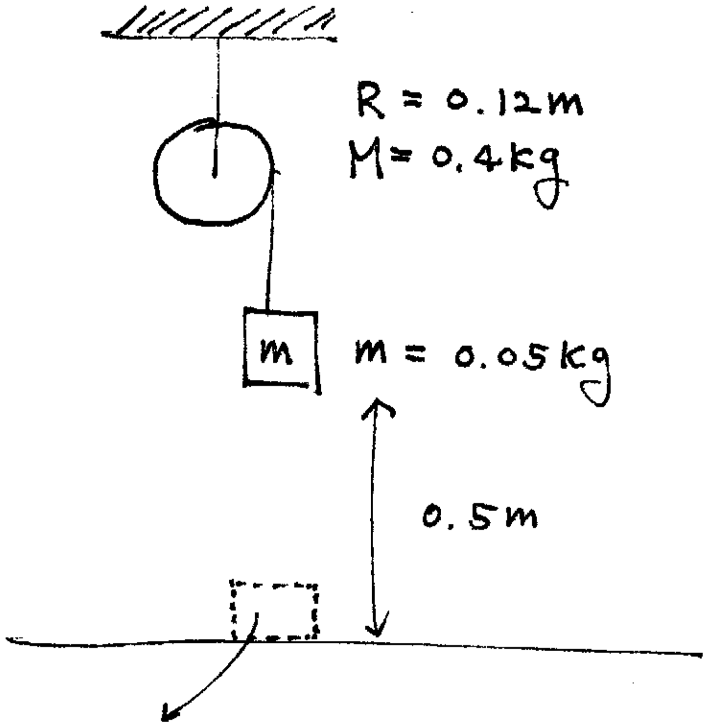
6. The pulley shown has a radius R and moment of inertia I. One end of the mass m is connected to a spring of force constant k, and the other end is fastened to a cord wrapped around the pulley. *Assume:* The pulley axle and the incline are frictionless. If the pulley is wound counterclockwise so as to stretch the spring a distance d from its equilibrium position and then released from rest, find the angular speed of the pulley when the spring is again unstretched; (*i.e.*, at the spring's equilibrium position). Answer in units of rad/s.



($\sqrt{\frac{2mg\bullet d\sin\theta + kd^2 - mv^2}{I}}$)

W6H6

1.



$I = \frac{1}{2} MR^2 = 0.00288$

$\Rightarrow$ P.E. = mgh = $0.05 \times 10 \times 0.5$

K.E. = $\emptyset$

$\therefore$ T.E. = 0.25 J.

P.E. = $\emptyset$

K.E. = $\frac{1}{2} m v^2 = \frac{1}{2}(0.05) v^2$

$KE_{DISK} = \frac{1}{2} I \omega^2 = \frac{1}{2} I \cdot \left(\frac{v}{R}\right)^2 = \frac{1}{2} I \cdot \frac{1}{R^2} v^2$

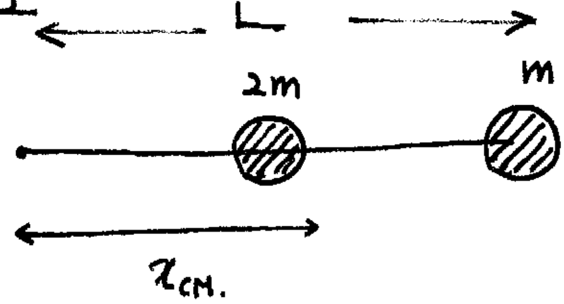
$\therefore 0.25 = 0.025 \cdot v^2 + 0.1 v^2$

$\therefore 0.125 v^2 = 0.25$

$v = \sqrt{\frac{0.25}{0.125}} = \sqrt{2} = 1.414 \ m/s$

W 6H-6

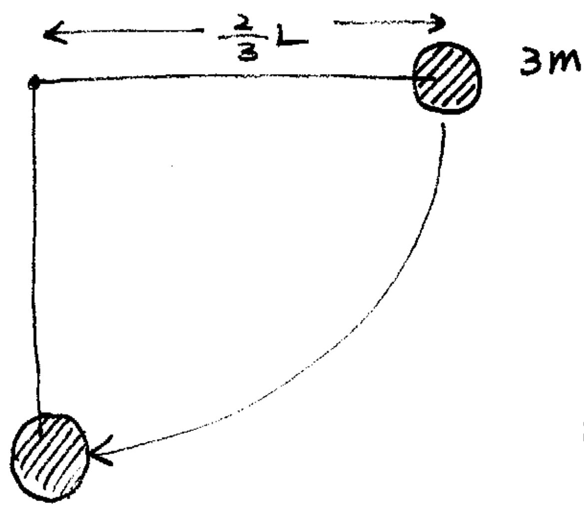
2.



$$3m\, x_{CM} = \cancel{2}m \cdot \frac{L}{\cancel{2}} + m \cdot L$$

$$x_{CM} = \frac{2}{3}L$$

$$I = 2m\left(\frac{L}{2}\right)^2 + m\left(L\right)^2 = \frac{m}{2}L^2 + mL^2 = \frac{3}{2}mL^2$$



$$P.E. = 3mg \cdot \left(\frac{2}{3}L\right) = 2mgL$$

$$K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2}\,\frac{3}{2}mL^2 \cdot \omega^2 = \frac{3}{4}mL^2\omega^2$$

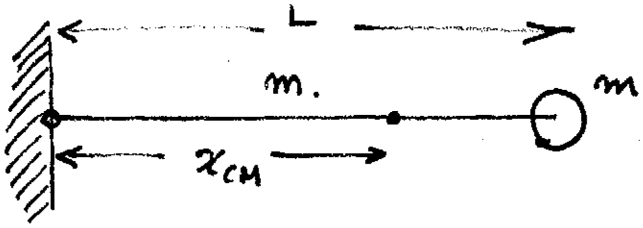
$$\therefore \quad 2\cancel{m}gL = \frac{3}{4}\cancel{m}L^{\cancel{2}}\omega^2$$

$$\omega^2 = \frac{8g}{3L}$$

$$\omega = \sqrt{\frac{8g}{3L}}$$

U6H6.

3.



$$I_{rod} = \frac{1}{3} m L^2$$

(a) $I = I_{rod} + m \cdot L^2 = \frac{1}{3} m L^2 + m L^2 = \frac{4}{3} m L^2$

(b). $2m \cdot x_{CM} = m \cdot \frac{L}{2} + m \cdot L = \frac{3}{2} m L$

$$x_{CM} = \frac{3}{4} L$$

(c). $2mg \left(\frac{3}{4} L\right) = \frac{1}{2} \left(\frac{4}{3} m L^2\right) \omega^2$

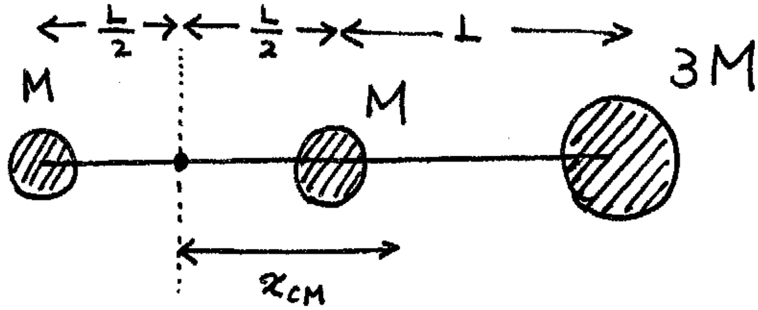
$$\frac{3}{2} \cancel{m} g \cancel{L} = \frac{2}{3} \cancel{m} L^{\cancel{2}} \omega^2$$

$$\omega^2 = \frac{9g}{4L}$$

$$\omega = \sqrt{\frac{9g}{4L}}$$

U6H6

4.



$$5M \cdot x_{CM} = M\left(\cancel{-\frac{L}{2}}\right) + \cancel{M\left(\frac{L}{2}\right)} + 3M\left(\frac{3}{2}L\right)$$

$$x_{CM} = \frac{9}{10}L$$

$$I = M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 + 3M\left(\frac{3}{2}L\right)^2$$

$$= \left(\frac{1}{4} + \frac{1}{4} + \frac{27}{4}\right)ML^2$$

$$= \frac{29}{4}ML^2$$

$$(\cancel{5}M)g\left(\frac{9}{\cancel{10}_{2}}L\right) = \frac{1}{2}\left(\frac{29}{4}ML^2\right)\omega^2$$

$$\frac{9}{2}\cancel{M}g\cancel{L} = \frac{29}{8}\cancel{M}L^{\cancel{2}} \cdot \omega^2$$

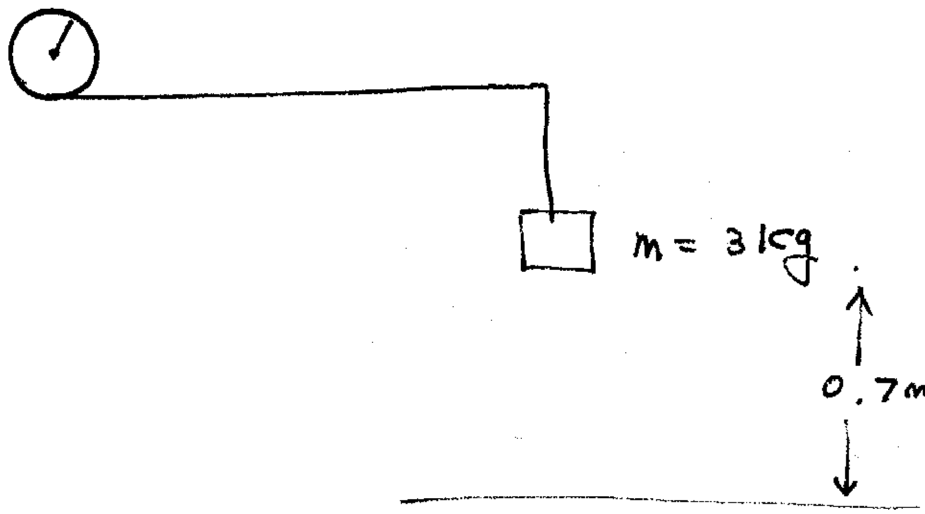
$$\omega^2 = \frac{9 \cdot \cancel{8}^{4} g}{\cancel{2} \cdot 29 L}$$

$$\omega = \sqrt{\frac{36 g}{29 L}}$$

W6H6

5.

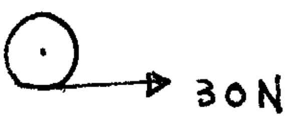
$M = 1.3\,kg$
$R = 0.02\,m$



$t = 20$ sec

$y = v_0 t + \frac{1}{2} a t^2$.

$0.7 = \frac{1}{2} \cdot a \cdot 20^2 \longrightarrow a = 0.0035\ m/s$



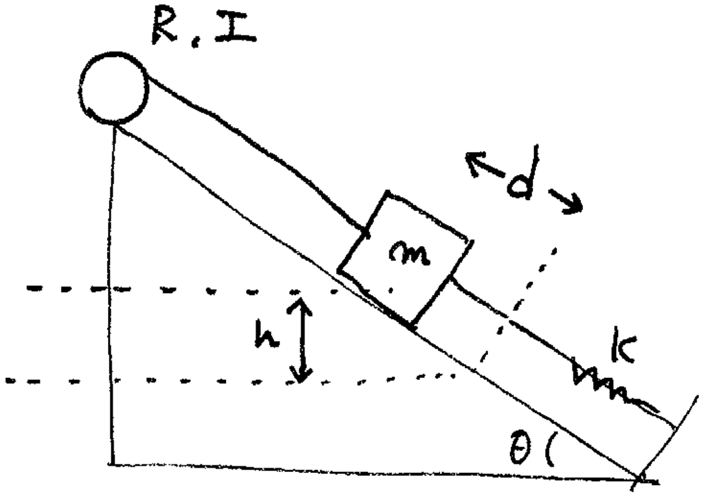
$\therefore \tau = 30 \times 0.02 = I \cdot \alpha$

$\alpha = \frac{a}{R} = \frac{0.0035}{0.02} = 0.175$

$\therefore I = \frac{30 \times 0.02}{0.175} = 3.43\ kgm^2$

U6H6

6.

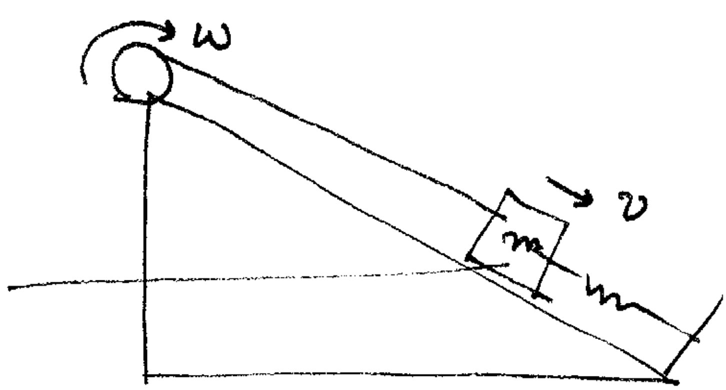


$$P.E. = mgh = m \cdot g \cdot d \cdot \sin\theta .$$

$$P.E._{spring} = \frac{1}{2} k d^2$$

$$K.E. = 0.$$

$$T.E. = mg \cdot d \cdot \sin\theta + \frac{1}{2} k d^2$$



$$P.E. = 0$$

$$K.E. = \frac{1}{2} m v^2 = \frac{1}{2}(m)(\omega R)^2 = \frac{1}{2} m R^2 \cdot \omega^2$$

$$K.E. = \frac{1}{2} I \omega^2$$

$$T.E. = \left(\frac{1}{2} m R^2 + \frac{1}{2} I\right) \omega^2$$

$$\therefore \quad mgd\sin\theta + \frac{1}{2} k d^2 = \left(\frac{1}{2} m R^2 + \frac{1}{2} I\right) \omega^2$$

$$\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{mR^2 + I}}$$